On the economic justification for the Loss of Load Expectation in the presence of Energy Limited Resources

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ABSTRACT

The Loss of Load Expectation (LOLE) of a power system, that is the expected number of hours per year in which it is unable to satisfy its load, is justified economically for EU power systems despite the presence of Energy Limited Resources (ELRs) such as battery storage systems casting uncertainty on whether such a justification is correct. We illustrate that the validity of this economic justification requires ELRs to minimise the depth of load shedding during scarcity events. We prove analytically that this operation is enforced if the marginal cost of shedding load is a convex and strictly increasing function and validate this hypothesis using a capacity expansion planning model. We suggest that an economic justification for a LOLE target requires this operation of ELRs to be consistently applied in adequacy assessments and capacity accreditation to ensure optimal system planning.

Keywords: Resource adequacy, Energy limited resources, Reliability standard, Loss of Load Expectation, Electricity storage

ACRONYMS

EENS Expected Energy Not Served

LOLE Loss of Load Expectation

VRES Variable Renewable Energy Sources

CONE Cost of New Entry

VOLL Value of Lost Load

ELR Energy Limited Resource

EU European Union

ERAA European Resource Adequacy Assessment

ENTSO-E European Network of Transmission System Operators - Electricity

ACER Association for the Cooperation of Energy Regulators

1. INTRODUCTION AND MOTIVATION

A commonly used resource adequacy metric is the Loss of Load Expectation (LOLE), which is the expected number of hours (or days) per year when there is insufficient capacity to meet load. It is the metric of choice for at least 11 power systems in Europe and North America (National Grid, 2017). In addition, since 2020 all European Member States are required to use the LOLE (ACER, 2020a).

Despite it's widespread use, employing the LOLE as a resource adequacy metric in isolation is widely criticised. A non-exhaustive list of these criticisms inspired by Stenclik et al. (2021) includes a lack of information on the duration of a scarcity event, since 10 scarcity events of one hour result in the same Loss of Load Expectation as one 10 hour scarcity event; a lack of information on the magnitude of scarcity events; it only provides an expected value, with no information on the distribution; and it is inconsistently defined across jurisdictions. These criticisms may also apply to other metrics applied in isolation, and indeed Stenclik et al. (2021) argue to use a variety of metrics so as to better determine the what type of resource (e.g. gas fired turbine, storage or demand response) should be added to improve adequacy at the lowest cost.

Stenclik et al. (2021) also argue that a resource adequacy metric should be 'transparent and economic'. Ironically, one of the arguments for using the LOLE is that it results quite naturally from solving a capacity expansion planning problem, where it is given by the ratio of the Cost of New Entry (CONE) and an expected Value of Lost Load (VOLL)[1]:

$$LOLE = \frac{CONE}{VOLL} \tag{1}$$

[1]See e.g. De Vries (2004, Section 5.4.2), though many other examples exist.

This economic argument is made explicit in a methodology by Association for the Cooperation of Energy Regulators (ACER) (ACER, 2020a) which stipulates that the LOLE should be calculated in this way for all European Member States.

This textbook economic argument is complicated by the presence of Energy Limited Resources (ELRs) such as battery storage systems, a matter discussed at length by Zachary et al. (2022) and acknowledged by ACER (2020a). The former argues that the relationship between LOLE, CONE and VOLL cannot be made in the presence of storage while the latter argues this justification holds if 'energy constraints are properly represented through the de-rating capacity factor'. To the best of our knowledge, no other literature has adressed this issue since despite the ever-growing amount of ELRs in modern power systems.

The principal contribution of this paper is then to clarify under what conditions a LOLE target can be justified economically for power systems with ELRs. More concretely, we answer the following research questions:

1. Is the economic justification for a LOLE target correct in the presence of ELRs? Yes, depending on how the ELRs is operated. We prove analytically that if the marginal cost of load shedding is convex and strictly increasing then ELRs are operated so as to minimise the depth of load shedding and the justification is correct. We further validate these results in a capacity expansion planning model. This numerical model also confirmed our findings despite relaxing simplifying assumptions made in the analytical model, which only considers a single ELR and ignores complicating constraints such roundtrip efficiencies or intertemporal losses.

2. Must the equation relating the LOLE to the CONE and expected VOLL be modified in some way to achieve this? No, though we highlight the importance of the term in this equation which can be interpreted as the inframarginal, non-scarcity rents of the marginal resource. In our numerical do validation, neglecting this term alters the analytical LOLE by 0 to 1 $\mathrm{h\,yr^{-1}}$.

3. Does ignoring the issues raised by ELRs lead to a sufficiently 'wrong' LOLE for this to be a cause for concern? We argue that it does. We base this assertion on our numerical validation, in which we found that the analytical and numerical LOLE disagreed by 2.5 $\mathrm{h\,yr^{-1}}$, as well as a previous work (Gonzato et al., 2023) where we found that the LOLE ranged from 2 to 6 $\mathrm{h\,yr^{-1}}$ depending on the operation of short term storage. We further argue that in resource adequacy related modeling exercises ELRs should be operated so as to minimise load shedding depth to ensure a coherent adequacy framework and an optimally planned power system.

We also qualitatively discuss the analogies with the work of Astier and Ovaere (2022) on the optimal LOLE in interconnected power systems.

These research questions are particularly important in the context of increasing Variable Renewable Energy Sourcess (VRESs) which in turn incentivises investments in ELRs such as batteries (Cebulla et al., 2018). It is natural to assume that the impact of an incorrect economic justification for a LOLE target will increase as the penetration of ELRs increases. In addition there is a substantial cost associated with a sub-optimal level of adequacy, that is a LOLE which is too great or too small. In recent years there has been a proliferation of capacity remuneration mechanisms implemented by European Member States to ensure enough capacity to satisfy resource adequacy needs. These are projected to cost seven billion Euros per year by 2023 (ACER, 2022b). Since the level of capacity procured and therefore cost of these mechanisms is driven by the target level of adequacy, a correct calculation of the socially optimal LOLE, which we investigate here, should be of particular interest to governments, system operators and regulators, particularly in the European Union.

The story and structure of this paper is illustrated in Figure 1. We begin by showing how the economic justification for LOLE may break down in the presence of ELRs depending on how it is operated in Section 2. In Section 3, we use the optimality conditions of a simplified ELR dispatch problem to show that a convex and strictly increasing marginal cost of load shedding leads to the desired ELR operation. We confirm this result using a capacity expansion planning model in Section 4. We finish with a comparison of our results with those in interconnected systems and a discussion on policy implications in Section 5.

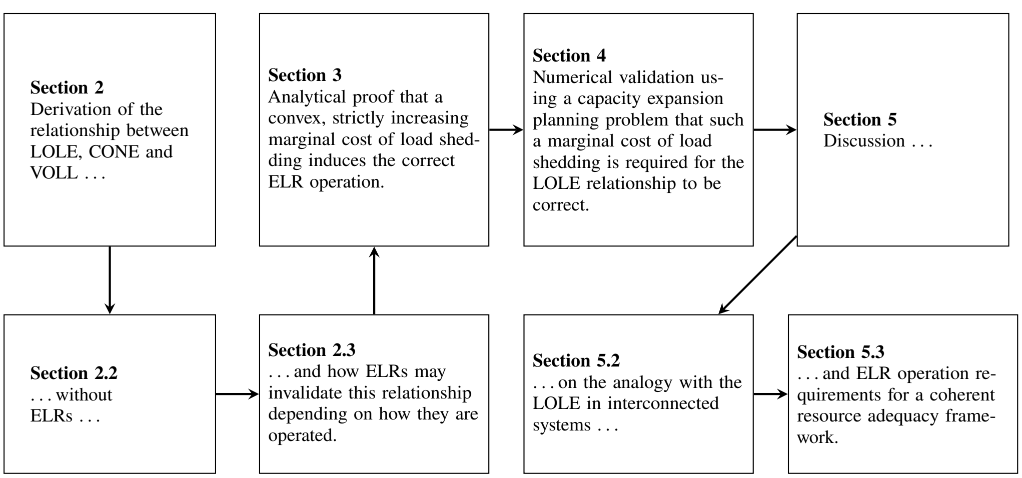

Figure 1: Structure or storyline of this paper.

2. ECONOMIC JUSTIFICATION FOR LOLE

In this section we will derive the textbook relationship between LOLE, CONE and VOLL by solving a cost minimisation problem which balances the cost of additional capacity (or adequacy) with the cost of shedding load. We will show that a central assumption when deriving this relationship, namely that adding a marginal amount of firm capacity reduces load shedding equally during scarcity timesteps, breaks down for a 'greedy' ELR operation. It does not with a load shedding depth minimisation operation. This motivates Section 3, where we prove that a convex, strictly increasing marginal cost of load shedding enforces such an ELR operation and so 'restores' the validity of the LOLE equation which we derive here.

2.1 The cost minimisation problem

Consider a set of resources $\mathcal{R}$ which make up a power system and a set of scenarios $s \in \mathcal{S}$ with associated weights ω_s and hourly timesteps $t \in \mathcal{T}$[2]. The fixed investment cost of these resources is $C^{fix}(\mathcal{R})$. The variable cost of operating or dispatching these resources is $C^{var}_{st}(\mathcal{R})$. The cost of shedding load is $W(\phi_{st}(\mathcal{R}))$, with the dependence of ϕ_{st} on $\mathcal{R}$ omitted from now on for the sake of brevity. The resulting cost minimisation problem can be written as follows:

$$\min \quad \sum_{s\in\mathcal{S}} \omega_s \cdot \left(\sum_{t\in\mathcal{T}} W(\phi_{st}(\mathcal{R})) + C^{var}_{st}(\mathcal{R}) \right) + C^{fix}(\mathcal{R}) \tag{2}$$

Additionally let:

$$W(\phi_t) = \int_0^{\phi_t} w(\phi) d\phi \tag{3}$$

The relationship between W and w is grahically illustrated in Figure 2. Two definitions of w are given, the first where it is a constant value and a second in which it linearly increases and then becomes constant again. These two definitions are used in the numerical study in Section 4 and we shall see that the latter is required for the economic justification of LOLE to be correct.

While it is not typical to assume that $w(\phi)$ increases linearly with ϕ, assuming such a functional form may be justifiable if load can be shed so as to cut off consumers with a progressively increasing VOLL. For example Belgium's emergency load shedding plan is performed in 'tranches', with low

[2]The assumption of hourly timesteps is not necessary but is made to simplify notation.

priority tranches (presumably composed of consumers with a low VOLL) being cut off first (Elia, 2019). Introducing such tranches for individual consumers for even greater granularity of their VOLL has also been investigated in the literature (Mou et al., 2020).

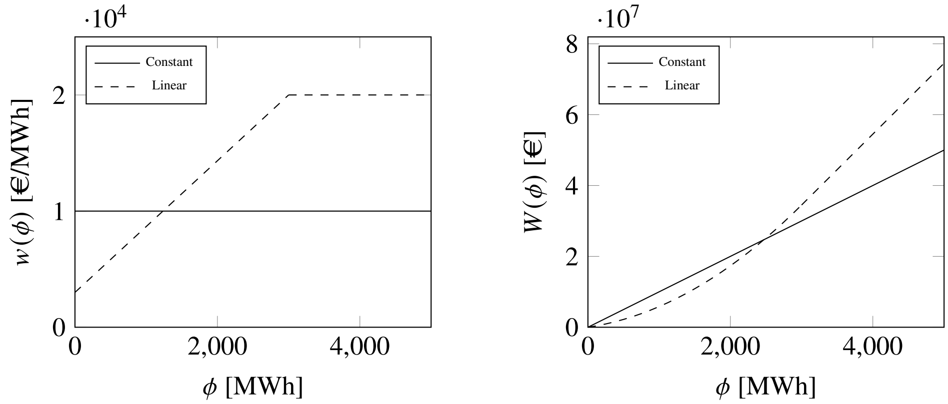

Figure 2: Marginal (left) and total (right) cost of load shedding as a function of the depth of load shedding.

2.2 Optimality criterion for a resource mix composed of firm and variable generation

In this section we will derive the optimality criterion for a capacity expansion planning problem which is composed only of firm and variable generation. This result, which will relate the optimal LOLE to the ratio of the CONE to the expected VOLL, is well established in the literature (De Vries, 2004, Section 5.4.2).

Following the methodology of Zachary et al. (2022), the optimal point of Eq. (2) is obtained by taking the derivative w.r.t. to firm capacity and setting the resulting expression to 0. To simplify notation, we will distinguish between scarcity and non-scarcity events with the following sets:

$$Q^S = \{(s,t) | s \in \mathcal{S} \text{ and } t \in \mathcal{T} \text{ and } \phi_{st} > 0\} \tag{4}$$

$$Q^{NS} = \{(s,t) | s \in \mathcal{S} \text{ and } t \in \mathcal{T} \text{ and } \phi_{st} = 0\} \tag{5}$$

Let lower case letters denote the derivatives of variables w.r.t. to firm capacity. The optimality criterion is then:

$$\sum_{s,t \in Q^S} \omega_s \cdot \left(c_{rt}^{var}(\mathcal{R}) - w(\phi_{st})\right) + \sum_{s,t \in Q^{NS}} \omega_s \cdot c_{rt}^{var}(\mathcal{R}) + c^{fix}(\mathcal{R}) = 0 \tag{6}$$

The derivative w.r.t. to firm capacity of $W(\phi_{st})$ is $-w(\phi_{st})$ by definition. This result follows from noting that the derivative w.r.t. to firm capacity of ϕ'_{st} is -1 h, also by definition (see Appendix 8.1). In other words, adding one MW of firm capacity reduces load shedding or energy not served ϕ_{st} by one MW h during scarcity timesteps.

The sensitivity of variable costs w.r.t. to a marginal increase of firm capacity, that is the marginal change in variable cost $c^{var}_{rt}(\mathcal{R})$, is positive during scarcity and typically zero in non-scarcity events. During scarcity events, an additional amount of firm capacity will reduce the cost of shedding load by $-w(\phi_{st})$ and increase marginal variable costs by $c^{var}_{rt}(\mathcal{R})$.

An additional MW of marginal firm capacity does not change the optimal dispatch, and so the change in marginal cost of operating the power system during non-scarcity timesteps, $c^{var}_{rt}(\mathcal{R})\forall t \in Q^{NS}$ is typically zero. Under certain conditions, such as an elastic demand, it may be non-zero. This may seem unintuitive if one does not realise that the firm capacity is being added at the margin, that is at the end of the merit order. We discuss this in greater detail at the end of this section and also in Appendix 8.4.

The sensitivity of fixed costs w.r.t. to a marginal increase of firm capacity $c^{fix}(\mathcal{R})$ is positive. It can be interpreted as the the fixed cost of new entry, $CONE^{fix}$.

The summation over scarcity events in Eq. (6) can be rewritten as $LOLE \cdot (\overline{VOLL} - CONE^{var})$. Put differently, this expression can be rewritten as the number of scarcity timesteps per year, $LOLE$, multiplied by the difference between the mean marginal cost of load shedding during scarcity, $\overline{VOLL}$ and the mean marginal variable cost $CONE^{var}$ (see Appendix 8.2 for a derivation). $\overline{VOLL}$ depends on the optimal set of resources $\mathcal{R}^*$ and can be interpreted as the mean cost per MW h of shedding load of the power system obtained by solving Problem (2).

For brevity, we will denote the summation over non-scarcity events (the second term in Eq. (6) as x. We will discuss this term in greater detail at the end of this section.

We can now write the well known expression for an economically optimal LOLE:

$$LOLE = \frac{CONE^{fix} - x}{\overline{VOLL} - CONE^{var}} \tag{7}$$

Zachary et al. (2022) cite the additional requirement that there must be a one-one correspondence between the Expected Energy Not Served (EENS) (given by Eq. (8)) and the LOLE in order for Eq. (7) to be a sufficient criterion to specify the optimal resource mix.

[2]This derivative has units of time, in this case hours, which we will typically omit for brevity.

$$EENS = \sum_{Q} \omega_s \cdot \phi_{st} \tag{8}$$

In the following paragraphs we will comment on circularity issues and the interpretation of the variable x. While we make reference to an ACER methodology (ACER, 2020a), our comments are relevant for any power system which uses an economic justification for a LOLE target.

Assuming $w(\phi_{st}) = V$, $x = 0$ and $CONE^{fix}$ and $CONE^{var}$ are the de-rated fixed and variable costs of the marginal resource respectively, then the inputs to Eq. (7) are known without knowledge of the optimal set of resources $\mathcal{R}$. However, it is likely that w depends on the type of scarcity events the power system in question will encounter at the optimal point, and these scarcity events depend on the optimal set of resources $\mathcal{R}$. This circularity is noted in an ACER methodology (ACER, 2020a, Article 12), but only for $CONE^{fix}$, since the de-rated cost of capacity $CONE^{fix}$ depends on the availability of the new entrant (marginal firm capacity addition) during scarcity. We highlight that this issue is also present for $\overline{VOLL}$, which an ACER methodology (ACER, 2020a, Article 7) states should be determined based on surveys of consumer types and adjusted to take into account the "applicable load shedding process".

The variable x is also addressed in the methodology (ACER, 2020a, Annex 3), where x is the "non-negligible increase or reduction in costs ... other than fixed and variable costs related to ENS (energy not served) avoided" from an "additional capacity resource", which following the methodology of Zachary et al. (2022) is the marginal firm capacity addition. The examples given in ACER (2020a) of how this x parameter may be non-zero include participation in ancillary services or other sectors such as heat.

The variable x may also be interpreted as the non-scarcity infra-marginal revenues of the marginal resource. This is illustrated in Appendix 8.3 by taking the optimality conditions of a capacity expansion planning model. If ELRs such as storage or demand response are present in the set $\mathcal{R}$, x may be non-zero if there are timesteps in which the electricity price is raised above the marginal cost of the marginal generator without load shedding occurring[3]. Another case where this may occur is if demand is elastic (see Appendix 8.4). We will see in Section 4 how this interpretation is crucial in ensuring the validity of Eq. (7).

Having established this well known relation for the optimal LOLE, in the next section we will show

[3]Electricity price formation in the case of storage is a complex process. The interested reader is referred to Mertens et al. (2021) and Sioshansi (2014).

how it can break down in the presence of ELRs.

2.3 Optimality criterion when the resource mix includes Energy Limited Resource

Eq. (7) hinges on the result that $\phi'_{st} = -1$. The derivation of Eq. (7) when ELR is present is the same as before, except that it may no longer be the case that $\phi'_{st} = -1$ depending on how ELR is operated. Figure 3 illustrates how this may occur[4] for the case where $W(\phi_t) = V \cdot \phi_t$ and hence ELR operation is imposed (see Gonzato et al. (2023) and also Section 3). The ELR is being operated in a 'greedy' fashion, by discharging as much and as fast possible until its energy content is depleted. Adding k MW of firm capacity reduces ϕ_3 by $2k$ MW h, since k MW h of the store can now be shifted to $t = 3$. In this case the derivative of ϕ_3 w.r.t. firm capacity is -2 hours and not -1 hour.

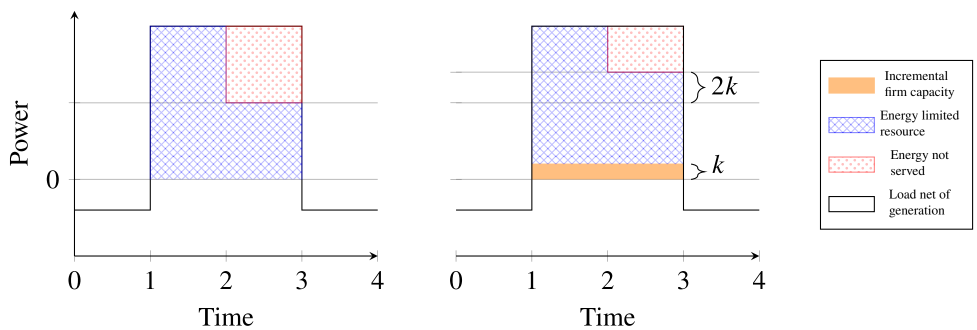

Figure 3: Illustration of how $\phi'_t \neq -1$ in the presence of ELR if a greedy operation is assumed. The addition of k MW of firm capacity reduces the amount of load shedding by $2k$ MW h and there is only scarcity in hour 3, hence the derivative ϕ'_t is -2 hours and not -1 hour as required.

However, this is not always the case. For the particular case of storage, Cruise and Zachary (2018) demonstrate that $\phi'_t = -1$ when ignoring stores which are depleted by the end of the scarcity event[5], i.e. stores which are typically of shorter duration or more energy limited. Ignoring these stores could be similar to imposing that they operate to minimise the severity of a scarcity event, since this particular operation means that such stores either completely eliminate a shortfall event or merely reduce its severity (the maximum value of ϕ_{st}) without reducing the number of scarcity hours.

Figure 4 illustrates that when this severity or load shedding depth minimising operation of ELRs is assumed then $\phi'_t = -1$ again. Adding k MW of firm capacity reduces load shedding by $2k$ MW h as

[4]This example is specific to ELR, such as batteries. In the case that such resources are power limited during a scarcity event, this not an issue (Cruise and Zachary, 2018).

[5]As highlighted by Cruise and Zachary (2018), the set of stores which are depleted by the end of the scarcity event is itself a random variable, since it depends on the nature of the scarcity event.

before, but the number of scarcity hours is 2, hence the derivative is -1 hour as required.

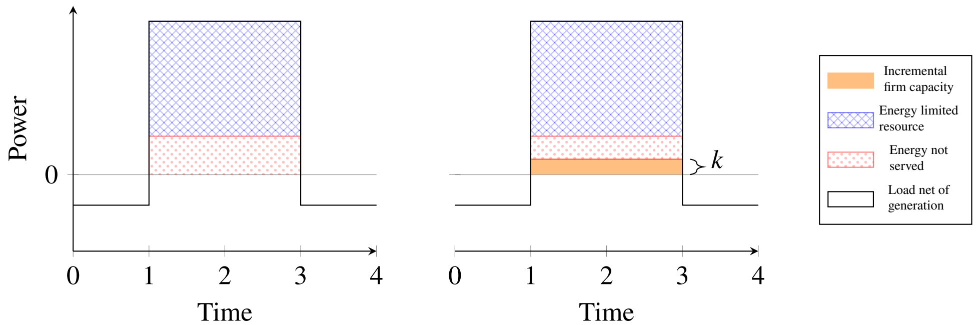

Figure 4: Illustration of how $\phi'_t = -1$ in the presence of ELR if a severity minimisation operation is assumed. The addition of k MW of firm capacity reduces the amount of load shedding by $2k$ MW h and there are two scarcity hours, hence the derivative is -1 hour as required.

In summary, it would appear that it is possible for the derivative of energy not served w.r.t. to firm capacity ϕ'_{st} to be equal to -1 in the presence of ELR, though this depends on how the storage is operated. In the following section we will present a stylised model for the dispatch of an energy constrained resource during a scarcity event and derive the necessary condition for the the dispatch to satisfy the requirement on ϕ_{st}. We will show that if $w(\phi)$ is convex and strictly increasing in ϕ then this requirement is met.

3. ANALYTICAL INVESTIGATION OF THE OPTIMAL OPERATION OF AN ENERGY LIMITED RESOURCE DURING SCARCITY

3.1 A stylised model of an Energy Limited Resource dispatch during a single scarcity event

Consider a single scarcity event $t \in \mathcal{T}^E$ with $n_E = |\mathcal{T}^E|$ and a single ELR with energy limit E and power limit P. The only decision to make is when to dispatch the ELR and by how much on the time interval $\mathcal{T}^E$, with d_t as the energy dispatched by the ELR. The resource may be energy constrained over the course of this event and it may be power constrained for certain time steps $t \in T^{PC} \subseteq \mathcal{T}^E$. Given a demand net of generation timeseries Z_t and a marginal load shedding cost $w(\phi)$ which is convex and strictly increasing in ϕ, the resulting optimisation problem can be written as:

$$\min \sum_{t \in \mathcal{T}^E} \int_0^{\phi_t} w(\phi) d\phi$$

$$
\begin{aligned}
\text{s.t.} \\
& d_t + \phi_t - Z_t = 0 \quad t \in \mathcal{T}^E \quad (\lambda_t) \\
& \sum_{t \in \mathcal{T}^E} d_t - E \leq 0 \quad (\mu) \\
& d_t - P \leq 0 \quad t \in \mathcal{T}^E \quad (\alpha_t) \\
& -d_t \leq 0 \quad t \in \mathcal{T}^E \quad (\gamma_t) \\
& -\phi_t \leq 0 \quad t \in \mathcal{T}^E \quad (\epsilon_t)
\end{aligned}
\tag{9}
$$

The Greek letters in brackets are the dual variables associated with each constraint.

3.2 Optimality conditions

In order to prove that $\phi'_t = -1$ when $w(\phi)$ is convex and strictly increasing in ϕ we will use the optimality conditions of Problem (9)[6]. These consist of the combination of first order optimality conditions of the problem, Eq. (10), and the complementarity conditions listed in Table 1. See Appendix 8.5 for the derivation of these.

$$\mu + \alpha_t + \epsilon_t = w(\phi_t) + \gamma_t \tag{10}$$

Table 1: Complementarity conditions of Problem (9) and their implications.

Complementarity	Dual variable > 0
$\mu \perp \sum_{t \in \mathcal{T}^E} d_t - E$	ELR is energy constrained
$\alpha_t \perp d_t - P$	ELR is power constrained
$\gamma_t \perp -d_t$	ELR discharge is 0
$\epsilon_t \perp -\phi_t$	Load shedding is 0

The resource may either be energy constrained ($\mu > 0$) or not, and equally there may be timesteps in which it is power constrained ($\alpha_t > 0$) or not. In the following section we will only consider the case in which the energy constraint is binding and give an intuitive proof as to why $\phi'_t = -1$ when $w(\phi)$ is convex and strictly increasing in ϕ. A mathematical proof of this is given in Appendix 8.5 along with a treatment of the case where the ELR is also power constrained. This latter case is only of peripheral interest to us in this context. This is because when an ELR is power constrained (but not energy constrained) it acts as a conventional generator, in which case we already know that $\phi'_t = -1$.

[6]The condition of convexity is required in order to interpret the optimality conditions of Problem (9).

3.3 Intuitive proof that $\phi'_t = -1$ when $w(\phi)$ is convex and strictly increasing in ϕ

If we assume that ELR is energy limited ($\mu > 0$) and not power limited ($\alpha_t = 0$) and only consider timesteps in which it dispatches ($\gamma_t = 0$) then we are left with the following condition:

$$\mu = w(\phi_t) \tag{11}$$

The above condition implies that in time steps where ELR dispatches the amount of load shedding ϕ_t is the same. This implication follows from the definition of $w(\phi)$ as strictly increasing in ϕ and hence there is a one-to-one mapping between $w(\phi)$ and ϕ.

This is not true when the marginal cost of load shedding is constant, i.e. $w(\phi) = V$, implying that the ELR operation is non-unique for the following reasons. If $w(\phi) = V$, then when ELR dispatches and there is load shedding, $\mu = V$. The ELR must be dispatched in each timestep, since otherwise $\mu = V = V + \gamma_t$, an impossibility. However, no other conditions can be imposed on d_t or ϕ_t other than that they must sum up to Z_t. Hence the cost optimal dispatch of the ELR during scarcity for $w(\phi) = V$ is non-unique.

Returning to the case where $w(\phi)$ is strictly increasing in ϕ, we have established that in timesteps when ELR dispatches but not enough to eliminate load shedding then the amount of load shedding ϕ_t is the same. In all other timesteps ELR is not dispatched and so load shedding is equal to the demand net of generation, i.e. $\phi_t = Z_t$. We prove in Appendix 8.5 that the amount of load shedding in these timesteps is less than in timesteps when ELR is dispatched. This implies that dispatching ELR up to the point that no load shedding occurs is not optimal and so the number of scarcity timesteps n_E remains unchanged by the ELR dispatch..

We can use these observations to describe the optimal ELR dispatch as illustrated in Figure 5. Initially, the ELR should be dispatched during the timestep with the greatest amount of load shedding until either the ELR is depleted or the load shedding is the same as in the timestep with the second greatest amount of load shedding. The ELR then dispatches to reduce load shedding equally in these two timesteps until it is either depleted or the load shedding is the same as in the timestep with the third greatest amount of load shedding, and so on.

Consider then what happens if a small amount of firm capacity k is added. This firm capacity addition is small enough that the ELR is still unable to completely eliminate the scarcity event. In this case, adding this firm capacity does not alter the ELR dispatch. We can therefore take the ELR dispatch as

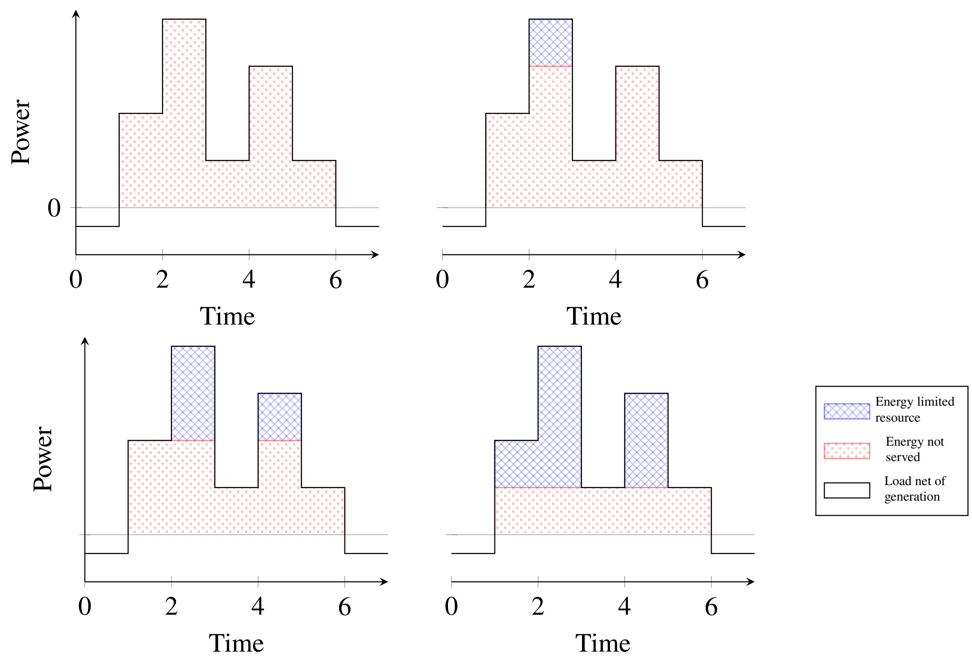

Figure 5: Illustration of ELR dispatch when $w(\phi)$ is convex and strictly increasing in ϕ. Figures should be read from top to bottom and left to right, with the ELR fully dispatched at the bottom right figure.

given and treat this as a scarcity event with a fixed demand net of generation where we include the net ELR dispatch in the generation. We know that for this case $\phi_t' = -1$, hence we obtain the result that we set out to prove.

3.4 Limitations to the analysis

The previous analysis assumed a single ELR with few dispatch constraints other than a power and energy limit. If the resource in question were storage, self-discharge losses would add constraints to Problem (9). Demand response may also require additional constraints to model how load can be shifted in time, as would modeling several ELRs. However, the dispatch problem of most ELRs in the face of a scarcity event is not fundamentally different to that described by Problem (9) and so it may be possible to extrapolate our central result, that $\phi_t' = -1$ when $w(\phi)$ is convex and strictly increasing in ϕ, to more complex ELRs types. To test the generalisability of our conclusions in our numerical study we relax the assumption of a single ELR and include roundtrip efficiencies and intertemporal losses.

4. NUMERICAL STUDY USING A CAPACITY EXPANSION PLANNING MODEL

In this section we will validate the analytical results of Section 2 using a numerical model. Specifically, we investigated whether a convex, strictly increasing marginal cost of load shedding $w(\phi)$ and correct calculation of the variable x results in agreement between the analytical LOLE, as defined by Eq. (7), and the numerical LOLE. All code and data can be found at https://gitlab.kuleuven.be/u0128861/storage-operation-and-planning.

4.1 Model and power system data

We use a static capacity expansion planning model to obtain an optimal resource portfolio which balances the marginal cost of shedding load with that of installing and operating resources. This model optimises the resource portfolio for a stylised, copperplate and islanded power system using Belgian timeseries data to numerically validate our analytical results. The capacity expansion is formulated as a linear program to avoid mismatches with our theoretical results due to non-convexities. This is the same model as that described in Appendix 8.3.

The resources invested in are limited to thermal generators, short term storage in the form of batteries and peak shaving demand response which is taken to be the marginal generator. We fix renewable capacities to those specified in Elia (2021) for the year 2032. This way we can determine scarcity days in advance and include them in the model in a manner similar to Hilbers et al. (2019) but without the need for an initial model run. We use 20 representative days, uniformly sampled from the maximum daily residual load duration curve, and 100 randomly sampled scarcity days (the top 5% of days on the curve) unless otherwise specified. This is illustrated in Figure 6.

The resource data is inspired by Elia (2021) and is summarised in Table 2. The Belgian load and VRES availability timeseries come from the Pan European Climate Database which used by among others European Network of Transmission System Operators - Electricity (ENTSO-E) in it's European Resource Adequacy Assessment (ERAA) (ENTSO-E, 2021).

Determining the LOLE using Eq. (7) requires calculating the expected marginal cost of shedding load, $\overline{VOLL}$. In the case where w is not constant, this can only be done once the planning and dispatch is known. $\overline{VOLL}$ is then calculated by taking the mean value of $w(\phi_{st})$ when $\phi_{st} > 0$.

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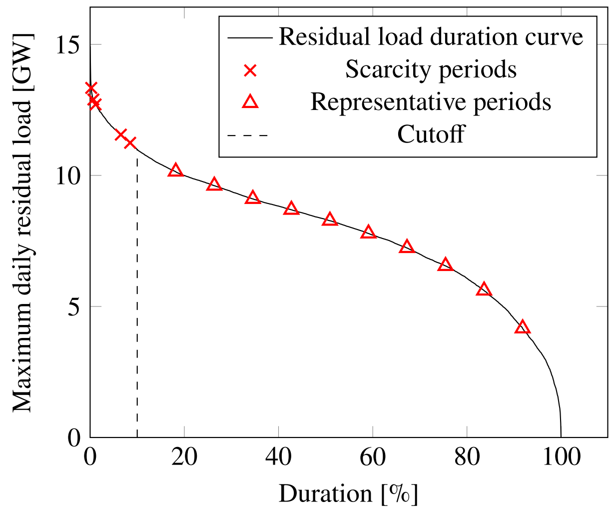

Figure 6: Illustration of the selection of 5 scarcity and 10 representative days, with scarcity days in the top 10% of the maximum daily residual load. The numerical model of Section 4 uses 20 representative days and 100 scarcity days.

known. $\overline{VOLL}$ is then calculated by taking the mean value of $w(\phi_{st})$ when $\phi_{st} > 0$.

Table 2: Summary of generation and storage technology parameters. CCGT = Combined Cycle Gas Turbine, OCGT = Open Cycle Gas Turbine, DR = Demand Response.

Technology	Fixed cost [€/kW/yr]	Variable cost [€/MWh]	Existing (maximum) capacity [GW]
Baseload	308.5	13.3	4.0 (4.0)
CCGT	73.2	139	0.0 (Inf)
OCGT	60.12	196	0.0 (Inf)
DR	50	500	0.0 (Inf)
Wind Onshore	104.0	0	5.4 (5.4)
Wind Offshore	176.0	0	4.4 (4.4)
Solar PV	66.26	0	12.2 (12.2)

	Fixed cost [€/kW/yr]	Variable cost [€/MWh]	Duration [h]	Round trip efficiency
Battery	56.4	0	2	0.9

4.2 How does Energy Limited Resource operation affect the agreement between the analytical and numerical LOLE?

We begin by investigating our primary assertion: that the ELR operation associated with a convex, strictly increasing marginal cost of load shedding $w(\phi)$ is necessary for the LOLE equation to be correct. We do this by varying the possibility of investing in batteries or not and the form of $w(\phi)$, either 'Constant' or 'Linear' in ϕ as illustrated in Figure 2. The results of this investigation are presented in Table 3.

In the absence of batteries there is no disagreement between the analytical and numerical LOLE, as

Table 3: If batteries can be invested in, $w(\phi)$ must be convex (here linear) in ϕ for the analytical and numerical LOLE to agree. LOLE values or differences in brackets are calculated without the factor x in Eq. (7). The function $w(\phi)$ is plotted in Figure 2

		LOLE [h yr^{-1}]				
Battery?	$w(\phi)$?	Numerical	Analytical		Difference	
✗	Constant	5.11	5.11	(5.26)	0	(0.15)
✗	Linear	11.50	11.50	(11.50)	0	(0)
✓	Constant	2.01	4.56	(5.26)	2.55	(3.25)
✓	Linear	11.13	11.13	(11.66)	0	(0.53)

expected from well established results in the literature. However, when batteries is present and $w(\phi)$ is constant, then this is not the case, which is in line with the claim of Zachary et al. (2022) since no battery operation is enforced. If $w(\phi)$ is linear then these two values agree again. This last result, which is proof that an economic justification for LOLE is possible in the presence of ELRs, is the principle contribution of this paper.

Some nuances should be noted. Firstly, the analytical LOLE for a linear $w(\phi)$ depends on whether storage is present or not. This perhaps surprising result is due to the different expected marginal cost of load shedding, i.e. $\overline{VOLL}$, in both cases. Put differently, the presence of storage affects the nature and therefore expected cost of load shedding events.

Secondly, including the factor x in the analytical calculation of LOLE eliminates the disagreement between the numerical and analytical LOLE in the case of a linear $w(\phi)$ and battery investment. While the error of excluding x is small (0.53 h yr^{-1}), this nuance is not mentioned in the methodology of ACER (2020a) which states that x is non-zero only in the presence of revenue streams other than those coming from the day ahead energy market. In the case of a constant $w(\phi)$ and no battery investment, this is due to a single timestep in which generation and load are exactly matched, that is to say that there is no scarcity, and so the electricity price[7] takes on a value between the variable cost of DR (500 €/MWh) and VOLL (10,000 €/MWh). The resulting error is small and could be also explained by the discrete representation of time in the model. In the case of a linear $w(\phi)$ and battery investment the term x is non-zero due to the battery raising electricity prices above the marginal cost of DR but below the VOLL as explained in Section 2.2. The price duration curves in Figure 7 illustrate these infra-marginal non-scarcity prices.

[7]We refer to the dual of the energy balance of our capacity expansion planning problem as the electricity price. Strictly speaking, when load shedding occurs this is not actually the electricity price but the marginal cost of shedding load. In reality, the electricity price would be capped in the day ahead market during scarcity moments. A more correct definition of this dual is then that it is the electricity price which would occur if all consumers could express their VOLL in the day ahead energy market.

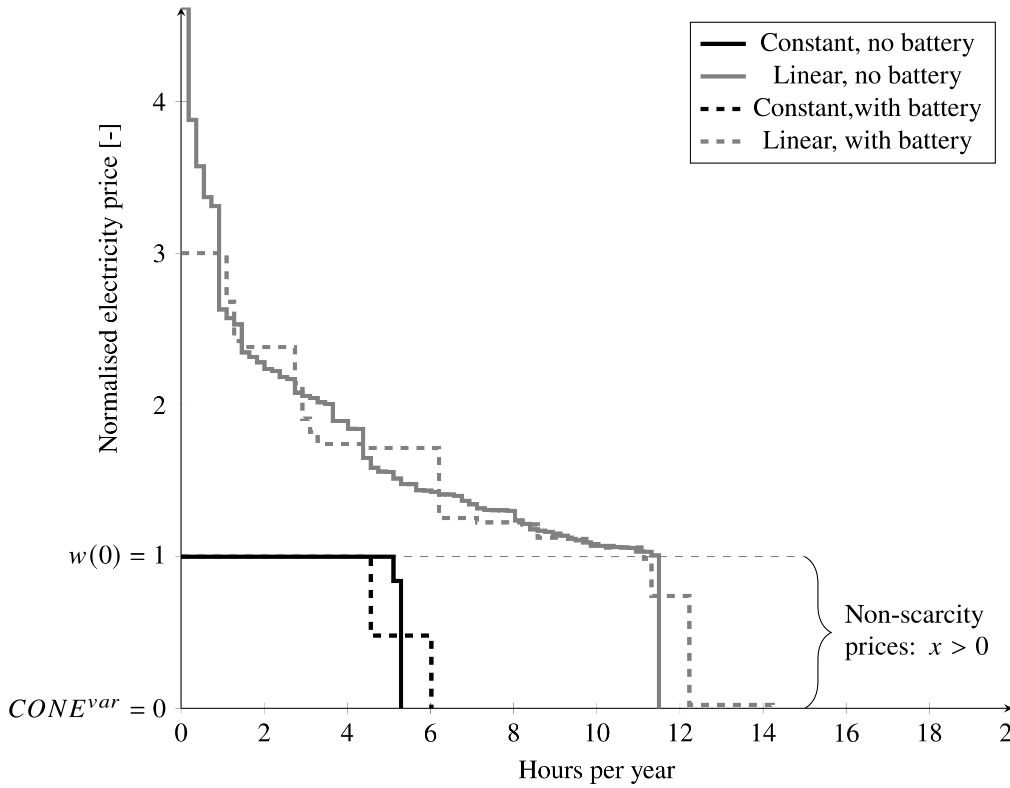

Figure 7: Non-scarcity infra-marginal prices are more common when batteries can be invested in than when they cannot. This gives a relatively greater weight to the factor x in Eq. (7). Prices have been normalised such that 0 is $CONE^{var}$ and 1 is $w(0)$ which is 10,000 €/MWh for 'Constant' and 3,000 €/MWh for 'Linear' (see Figure 2 for an illustration of these cases). Y-axis values between 0 and 1 are then infra-marginal, non-scarcity prices, while above 1 they are scarcity 'prices' which are set by the marginal cost of load shedding in that time step.

4.3 Are these results generalisable to other types of Energy Limited Resources?

The proofs of Section 2 and Section 3 were not limited to short term storage, raising the question of whether the agreement of the analytical and numerical LOLE holds more generally in the presence ELRs. We tentatively suggest that this is the case, assuming still that $w(\phi)$ is linear in ϕ, by looking at two additional cases - that of multiple storage units, and storage with losses.

In the case of multiple stores, 50 different stores could be invested in. Their characteristics were the same as that of the battery technology described in Table 2 except the duration could be between 1 and 4 hours, the fixed costs between 400 and 600 €/kW/yr and the round trip efficiency between 0.7 and 0.95. Values were randomly selected between these ranges for each store. The maximum capacity per store was limited to 0.5 GW to ensure that multiple stores were invested in.

In the case of storage which included losses, it was assumed that 5% of the stores energy content was lost between successive timesteps.

In both cases, the analytical and numerical LOLE agree to within 6 decimal places. This suggests that our results hold for ELRs more generally, irrespective of the particular energy related constraints. However, many other examples of ELRs are possible, such as demand response which can shift load in time, and this result should be interpreted with caution.

5. DISCUSSION

In this paper we have discussed how the presence of ELRs can invalidate the economic justification for LOLE. We showed that this justification assumes a particular operation of ELRs which can be induced by assuming a convex, strictly increasing marginal cost of shedding load, $w(\phi)$, first by illustration in Section 2 and then analytically in Section 3. We then validated this result numerically in Section 4 assuming a linear form of $w(\phi)$.

We will finish this paper by discussing limitations to our work in Section 5.1; compare these results to those in interconnected systems in Section 5.2; and comment on the implications of an economically justified LOLE target for adequacy assessments and capacity accreditation.

5.1 Limitations and future work

As we mentioned in Section 4, while our analytical results may be generalisable to any ELR, our numerical study was limited to looking at short term storage, specifically batteries. Future work should investigate whether our results hold for other ELR, such as demand shifting response similar to that considered in Zhou et al. (2015).

We have not considered here how a LOLE target could be economically justified if an ELR is the marginal unit or new entrant. A concrete example of such an ELR would be peak shaving demand response with a low maximum load factor i.e. an energy constraint defined on an annual basis of several $\mathrm{h\,yr^{-1}}$. Considering whether our results hold in this case may be of more than just academic interest, since such resources have been used to calculate LOLE in the past (CREG, 2021). An ACER methodology (ACER, 2020a) details how an economically justified LOLE should be adapted if a resource has a maximum load factor though without explaining the reasoning behind this. Future work could investigate this by expanding the analytical model of Section 3 to include multiple scarcity events and considering how the factor x would change if an ELR were the new entrant.

5.2 The economic justification for LOLE in interconnected systems: an analogy

One of the inspirations for this paper was the work of Astier and Ovaere (2022), who conducted a similar investigation into the economic justfication of LOLE for interconnected systems. In their case, the optimal LOLE for a particular country or market includes "hours where additional domestic capacity could have decreased lost load in the neighbor country" as well as "domestic" lost load hours.

This observation is remarkably similar to that of Zachary et al. (2022) (who cite the work of Cruise and Zachary (2018)) who claim that the optimal LOLE in the presence of storage is that which would occur ignoring the "set of stores which ... are empty at the end of the shortfall period." Indeed, it was this observation that prompted us to suggest that an economically justified LOLE is possible if ELRs are operated in such a way that they maximise LOLE, which emerges naturally in the face of a convex, strictly increasing marginal cost of load shedding.

A key insight of Astier and Ovaere (2022) was to differentiate between the optimal level of resource adequacy of a power system and its assessed adequacy[8]. We will build on this insight in the next section where we also add another ingredient to the mix, the realised adequacy of a power system.

[8]In Astier and Ovaere (2022) these are called the "simulated" and "realised" LOLE levels.

The similarities and differences of interconnectors and ELR in relation to resource adequacy are summarised in Table 4. The first observation is that in interconnectors allow distributing load shedding or energy not served in space while ELRs allow distributing it in time. If the marginal cost of shedding load is constant, i.e. $w(\phi) = V$, then there is no uniquely optimal way of distributing load shed in either space or time. We have investigated the latter case in a previous work (Gonzato et al., 2023) and proved it in Section 3. A similar situation occurs in interconnected systems. If two market zones or nodes have the same marginal cost of shedding load then cost minimisation cannot define where it is more optimal to shed load - any distribution of shed load between the two zones or nodes is cost optimal. In the case of ELR, this non-uniqueness means that any indicator other than EENS depends on the operation of ELR. For interconnected systems the non-uniqueness implies that the total EENS is well defined while the EENS in each market zone or node is not.

Defining an ELR operation or "load curtailment priority rule" (Astier and Ovaere, 2022) for interconnected systems solves this issue of non-uniqueness[9]. Alternatively, one can also differentiate $w(\phi)$ in time or space respectively to enforce a particular operation or load curtailment priority rule. This is implicitly what we did here, since by letting $w(\phi)$ be a convex, strictly increasing function in ϕ $w(\phi_{st})$ varies in time. When calculating the optimal LOLE using Eq. (7), the central result of this paper and that of Astier and Ovaere (2022) is remarkably similar: this optimal LOLE is the one which results from an ELR operation or "load curtailment priority rule" which maximises the LOLE in that market zone or node. For ELRs this is a load shedding depth minimising operation while for interconnectors this means prioritising EENS in that market zone or node.

A final difference between ELRs and interconnected systems is that installed capacities of resources can vary in space but not in time. Concretely, it is possible to install a wind turbine in, say, Belgium or the Netherlands but once it's installed the nominal capacity of the wind turbine does not change. We suggest that the implication of this is that, assuming $w(\phi) = V$, a load curtailment priority rule would affect the location of installed capacities while enforcing a particular ELR operation would not.

5.3 A consistent Energy Limited Resource operation is required in reliability standard calculations, adequacy assessments and capacity remuneration mechanisms

We emphasise that what follows is firmly grounded in the resource adequacy framework currently in place in the European Union (EU), as defined by the ACER methodologies ACER (2020a) and ACER

[9]However, unlike ELR operation, regulations exist in the EU stipulating what load curtailment priority rule should be used (Astier and Ovaere, 2022).

Table 4: Comparison of interconnectors and ELRs in relation to resource adequacy. Note how replacing 'space' with 'time' leads to similar statements.

Interconnectors	Energy Limited Resource
Load shedding can be distributed in space	Load shedding can be distributed in time
There is no uniquely optimal distribution of load shedding in space if the marginal cost of load shedding is constant	There is no unique distribution of load shedding in time if the marginal cost of load shedding is constant
Spatial differentiation of the marginal cost of load shedding would resolve the above	Temporal differentiation of the marginal cost of load shedding would resolve the above
The optimal LOLE is the one that would occur if a market zone prioritises external EENS	The optimal LOLE is the one that would occur ignoring the ELRs which do not contribute to reducing EENS
The nominal capacity of a resource can be varied in space	The nominal capacity of a resource cannot be varied in time

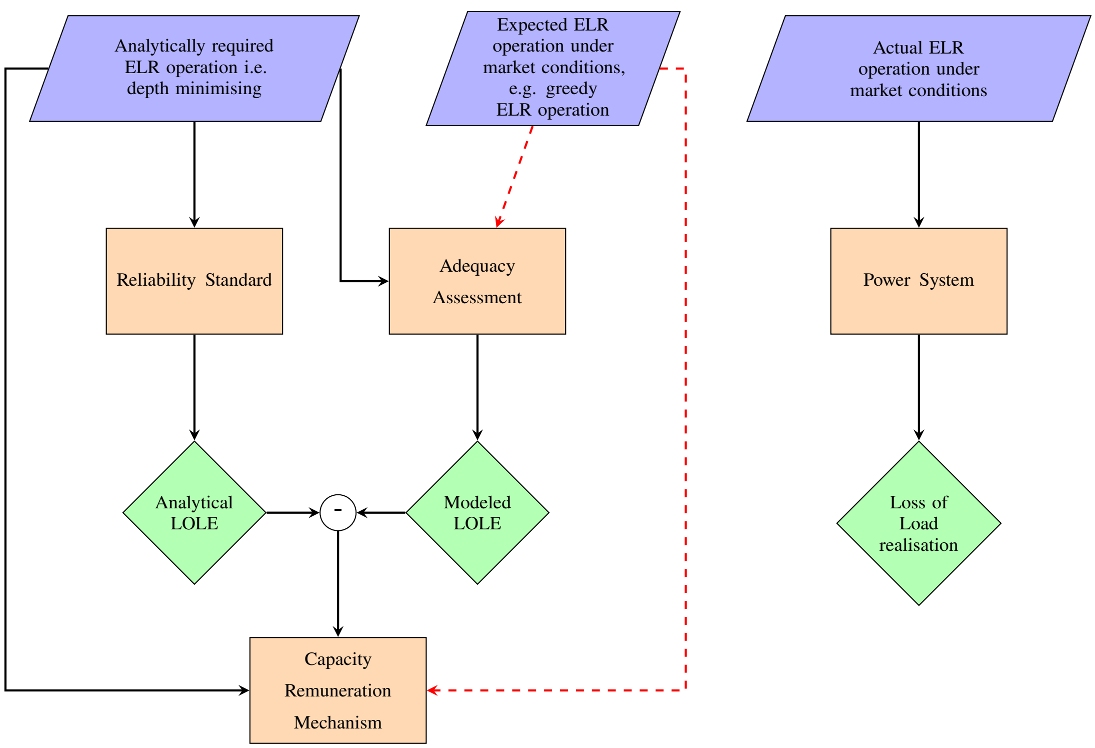

Figure 8: Stylised depiction of the EU's resource adequacy framework. Rectangles are modeling exercises or the physical power system, diamonds are the outputs of these and trapezoids are possible ELR operations. Black arrows are the flow of data which we believe to be correct while dashed red arrows are what is currently understood, though it is currently not widely appreciated that the reliability standard calculation presumes a particaular storage operation.

(2020b) among other regulatory and legal documents. We do not know of other jurisdictions or markets for which our paper is directly applicable, though the economic justification for a LOLE target is often invoked in the academic literature (Kaminski et al., 2023).

The resource adequacy framework in the EU is broadly composed of three parts: the setting of the target or optimal level of resource adequacy or reliability (LOLE) (ACER, 2020a), otherwise known as the reliability standard; adequacy assessments, specifically the ERAA (ACER, 2020b); and capacity remuneration mechanisms for the member states that have one[10]. Aside from this, there is also the realised adequacy of a power system, put differently the nature of realised scarcity events that occur within a market zone.

These parts and how they interact are illustrated in Figure 8. Broadly speaking, they interact as follows[11]. The optimal level of resource adequacy, as described by the LOLE, is calculated, e.g. by the national government and regulatory authority (ACER, 2022b, Table 3). The ERAA is then used to determine whether this LOLE target is satisfied in a market zone. If it is not, then under certain conditions implementing a capacity remuneration mechanism is justified so as to ensure sufficient resources are added to the power system to meet the LOLE target.

We have argued throughout this paper that a particular operation of ELR is required in order for the economic justification for the LOLE target to be valid. We further argue that this operation should be imposed on ELR when conducting the ERAA so that the target and assessment are consistent with each other. Modeling exercises similar to the ERAA are required to justify the implementation of a capacity remuneration mechanism, and so here too the same operation of ELR should be used.

This understanding of which ELR operation should be used and where is illustrated in Figure 8 (solid black arrows) along with the current understanding of what should be used (dashed red arrows). Crucially, a load shedding depth or severity minimising strategy should be used for all modeling exercises even if this is not representative of how ELRs would be operated under market conditions.

To understand why, consider the following example. A regulator calculates that the optimal LOLE is 3 h yr^{-1}, which, as we have shown here, assumes the severity minimising ELR operation illustrated in Figure 4. Then in the ERAA an LOLE of 3 h $\mathrm{yr}^{-1}$is calculated but assuming the greedy operation illustrated in Figure 3. If the severity minimising ELR were assumed then the LOLE would have been 6 h yr^{-1}. The power system is actually inadequate, but this was missed in the ERAA because of the inconsistent assumptions on the operation of ELR.

[10]We have omitted the complicating factor of the economic viability assessments conducted within adequacy assessments.
[11]For simplicity we will omit discussion of any circular dependencies such as those discussed in ACER (2020a).

A similar line of reasoning holds regarding capacity remuneration mechanisms. These mechanisms typically require determining the amount of additional capacity needed to reach a level of adequacy, in this case a LOLE target (Kaminski et al., 2023). If when conducting this exercise a greedy ELR operation is assumed (see Figure 3) then the amount of capacity required will be underestimated, since the greedy operation will produce a greater marginal capacity credit[12] of ELR than the severity minimising strategy assumed when setting the LOLE target (Gonzato et al., 2023; National Grid, 2017). The contribution of ELR to the capacity target is then over-estimated, potentially leading again to an inadequate system.

We highlight in passing that we are not alone in our concern with the consistency between the various parts of the resource adequacy framework. The Belgian energy regulator, the CREG, argues in a regulatory note that a capacity remuneration mechanism can only be coherent if the same reference technology (the new entrant) is consistently used to calculate parameters needed for the reliability standard and the capacity remuneration mechanism (CREG, 2022). Specifically they highlighted that for the Belgian capacity remuneration mechanism auction in 2021, demand side response was used as the reference technology when calculating the reliability standard while open cycle gas turbines were the reference technology for calculating the capacity remuneration mechanism's parameters such as the price ceiling.

A remaining question is whether the realised adequacy of the system should also be consistent with the modeling exercises in the resource adequacy framework. To simplify addressing this question, we will ignore many of the external factors, described in (Gonzato et al., 2023), which would affect an ELR operators decision making. In particular we will assume perfect foresight and that ELR participates in the electricity wholesale market.

The EU resource adequacy framework exists to ensure that the resource mix is as close to the 'optimal' social welfare maximising solution as possible. It could be argued then that the realised adequacy doesn't matter, as long as the resource mix is close to optimal. However, this ignores the possibility that the marginal cost of shedding load really is (to a good approximation) convex and strictly increasing, in which case any strategy other than a severity minimising one is sub-optimal from a social welfare perspective. In addition, different ELR strategies lead to different revenues for resources in the power system, since the number of hours at which the price cap is reached i.e. the LOLE would be different. This in turn leads to different investment incentives and therefore decisions, so that ultimately the

[12]The marginal capacity credit of a resource is the amount in MW of firm capacity that must be added to a baseline system such that it has the same adequacy as the baseline system plus an additional MW of that resource (Zachary et al., 2022).

choice of ELRs operation during scarcity may result in resource mixes that deviate from the optimal.

It is worth noting at this point that what is economically optimal may not be politically or socially acceptable. This issue has come to the fore recently in discussions regarding high energy prices and calls to change the wholesale market design in the EU (ACER, 2022a). We raise this point since a severity minimising operation of ELRs would lead to a greater number of hours in which the price cap is reached and so is unfavourable towards electricity consumers exposed to the wholesale market price. Regardless of this, our results still hold.

We will not elaborate further on the required market setup and ELR operation for investment incentives to align with resource adequacy framework as this is a non-trivial task. Indeed, the issue of coherency within the EU resource adequacy framework in general requires further investigation.

6. CONCLUSION

In this paper we investigated whether an economic justification for a LOLE target is possible in the presence of ELR. We concluded that it is possible but only if ELR is operating so as to minimise the severity or depth of load shedding. We proved this first analytically by investigating ELR dispatch for a stylised, single scarcity event and then validated this hypothesis numerically using a capacity expansion planning model. We also highlighted the importance of accounting for non-scarcity, inframarginal revenues of the marginal resource in the equation relating LOLE to the CONE and VOLL, sometimes called the reliability standard. Finally, we recommend that a severity minimising operation of ELR should be consistently employed in all modeling exercises within a resource adequacy framework to ensure that it is internally coherent and provide an optimal capacity target for capacity remuneration mechanisms.

7. ACKNOWLEDGEMENTS

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8. APPENDIX

8.1 Equivalent expressions for the marginal cost of shedding load

The required relationship is the following:

$$\frac{dW(\phi(k))}{dk} = -w(\phi(k)) \tag{12}$$

where k represents firm capacity, which is shown here to explicitly affect load shedding ϕ. Applying the chain rule we obtain the following:

$$\frac{dW(\phi(k))}{dk} = \frac{dW(\phi(k))}{d\phi} \cdot \frac{d\phi(k)}{dk} \tag{13}$$

This simplifies to:

$$\frac{dW(\phi(k))}{dk} = w(\phi(k)) \cdot \frac{d\phi(k)}{dk} \tag{14}$$

Comparing to Eq. (12), clearly it is required that $\frac{\phi(k)}{dk} = -1$, hence proving the former is equivalent to proving the latter. The intuitive interpretation of this result is that adding a MW of firm capacity reduces load shedding during scarcity time steps by a MW.

8.2 Variable and energy not served cost terms of Problem (2)

Consider the following expression:

$$\sum_{s,t \in Q^S} \omega_s \cdot \left(c_{rt}^{var}(\mathcal{R}) - w(\phi_{st})\right) \tag{15}$$

This bears resemblance to an expectation with weightings ω_s. The average number of scarcity hours per year is $LOLE = \sum_{s,t \in Q^S} \omega_s$. Making this explicit in Eq. (15):

$$\frac{LOLE}{LOLE} \cdot \sum_{s,t \in Q^S} \omega_s \cdot \left(c_{rt}^{var}(\mathcal{R}) - w(\phi_{st})\right) \tag{16}$$

$$LOLE \cdot \frac{\sum_{s,t \in Q^S} \omega_s \cdot \left(c_{rt}^{var}(\mathcal{R}) - w(\phi_{st})\right)}{LOLE} \tag{17}$$

$$LOLE \cdot \mathbb{E}\left[c_{rt}^{var}(\mathcal{R}) - w(\phi_{st})\right] \tag{18}$$

$$LOLE \cdot \left(\mathbb{E}[c_{rt}^{var}(\mathcal{R})] - \mathbb{E}[w(\phi_{st})]\right) \tag{19}$$

$$LOLE \cdot (CONE^{var} - \overline{VOLL}) \tag{20}$$

8.3 Formulation of a capacity expansion planning problem and derivation of its optimality conditions

We formulate our capacity expansion planning problem in standard form as:

$$\begin{aligned} \min \quad & \sum_{g \in \mathcal{G}} C_g^{fix} \cdot k_g + \sum_{h \in \mathcal{H}} C_h^{fix} \cdot k_h \\ & + \sum_{s \in \mathcal{S}, t \in \mathcal{T}} \omega_s \cdot \left(\sum_{g \in \mathcal{G}} C_g^{var} \cdot q_{gst} + W(\phi_{st}) \right) \end{aligned} \tag{21}$$

s.t.

$$\sum_{g \in \mathcal{G}} q_{gst} + \sum_{h \in \mathcal{H}} (d_{hst} - c_{hst}) + \phi_{st} - D_{st} = 0 \qquad s \in \mathcal{S},\ t \in \mathcal{T}\ (\lambda_{st}) \tag{22}$$

$$q_{gst} - AF_{gst} \cdot k_g \leq 0 \qquad g \in \mathcal{G},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (\mu_{gst}) \tag{23}$$

$$d_{hst} - k_h \leq 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (\gamma_{hst}) \tag{24}$$

$$c_{hst} - k_h \leq 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (\nu_{hst}) \tag{25}$$

$$e_{hst+1} - LF_h \cdot e_{hst} + \left(\frac{d_{hst}}{\eta_h} - c_{hst} \cdot \eta_h \right) \cdot \Delta_t = 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (\delta_{hst}) \tag{26}$$

$$e_{hst_1} - e_{hst_{end}} \leq 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S}\ (\chi_{hst}) \tag{27}$$

$$e_{hst} - P2E_h \cdot k_h \leq 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S}\ t \in \mathcal{T} (\epsilon_{hst}) \tag{28}$$

$$-q_{gst} \leq 0 \qquad g \in \mathcal{G},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (m_{gst}) \tag{29}$$

$$-k_g \leq 0 \qquad h \in \mathcal{H}\ (\alpha_h) \tag{30}$$

$$-d_{hst} \leq 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (o_{hst}) \tag{31}$$

$$-c_{hst} \leq 0 \qquad h \in \mathcal{H},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (n_{hst}) \tag{32}$$

$$-e_{hst} \leq 0 \qquad g \in \mathcal{G},\ s \in \mathcal{S},\ t \in \mathcal{T}\ (\sigma_{gst}) \tag{33}$$

$$-k_h \leq 0 \qquad h \in \mathcal{H}\ (\beta_h) \tag{34}$$

$$-\phi_{st} \leq 0 \qquad s \in \mathcal{S},\ t \in \mathcal{T}\ (\rho_{st}) \tag{35}$$

Since this capacity expansion planning problem is typical of many found in the literature (see for example a previous work of ours (Gonzato et al., 2021)) we will exhaustively describe its parameters, variables and constraints.

The first two terms in the objective function are the product of the costs per unit of installed generation or storage capacity, C_g^{fix} and C_h^{fix} respectively, and the installed capacities of generators and storage units, k_g and k_h respectively. The remaining terms are the variable costs, which are weighted by the scenario (a.k.a. representative period) weights ω_s. These are divided into the generation costs, $C_g^{var} \cdot q_{gst}$, and the load shedding costs, $W(\phi_{st})$ where the function W may be one of the two forms illustrated in Figure 2.

Proceeding chronologically, Constraint (22) is the energy balance which specifies that the amount generated plus the total net charging and discharging of storage $\sum_{h\in\mathcal{H}}(d_{hst} - c_{hst})$ plus the load shed must be equal to the demand D_{st} for all scenarios and timesteps. Constraint (23) limits the energy generated to the installed capacity of the generator derated by its availability factor AF_{gst}. Constraints (24) and (25) similarly limit the charging and discharging of storage units to their installed power capacity. Constraint (26) the energy balance for storage, which includes the efficiency loss parameter LF_h which was set to 1 except for in Section 4.3. η_h is the (dis)charging efficiency. The cyclic constraint (27) is imposed for each scenario and enforces the state of charge of storage at the end of a scenario to be greater than or equal to the initial state of charge. Constraint (28) limits the state of charge to the energy capacity of the storage unit, $P2E_h \cdot k_h$. The remaining constraints (29) - (35) ensure that all variables are non-negative.

It is straightforward to show that the optimality condition for the peaking generator, $\hat{g}$, can be written as follows:

$$C_{\hat{g}}^{fix} = \sum_{\substack{s\in\mathcal{S},t\in\mathcal{T}:\\ \lambda_{st}>C_{\hat{g}}^{var}}} AF_{\hat{g}st} \cdot \omega_s \cdot (\lambda_{st} - C_{\hat{g}}^{var}) \tag{36}$$

This expression, which holds for any generator, states that the peaking generator must recover it's fixed costs through the electricity price λ_{st}. More precisely, λ_{st} is the electricity price which would occur if consumers were able to express their VOLL in the day ahead market instead of having their demand curtailed.

Typically it is assumed that since we are dealing with the peaking generator, $\lambda_{st} > C_{\hat{g}}^{var}$ is a sufficient condition to describe a scarcity event. However, in some cases, e.g. in the presence of storage, the electricity price may be greater than the variable cost of the peaking generator without load shedding taking place. We can highlight this by seperating Eq. (36) into scarcity and non-scarcity events, using the same notation as in Section 2 and only considering cases in which $\lambda_{st} > C_{\hat{g}}^{var}$[13]:

$$C_{\hat{g}}^{fix} = \sum_{s,t \in Q^S} AF_{\hat{g}st} \cdot \omega_s \cdot (\lambda_{st} - C_{\hat{g}}^{var}) + \sum_{s,t \in Q^{NS}} AF_{\hat{g}st} \cdot \omega_s \cdot (\lambda_{st} - C_{\hat{g}}^{var}) \tag{37}$$

Comparing with Eq. (7), we can see that the variable x corresponds to the first summation in Eq. (37). In other words, x can be alternatively be interpreted as the non-scarcity infra-marginal revenues of the peaking generator. Since the duals and therefore λ of a capacity expansion planning problem are easily obtained, this interpretation of x is more easily applied than that which results from a cost minimisation. Indeed, we use this interpretation to calculate x in Section 4.

8.4 Non-scarcity infra-marginal revenues in the presence of an elastic demand

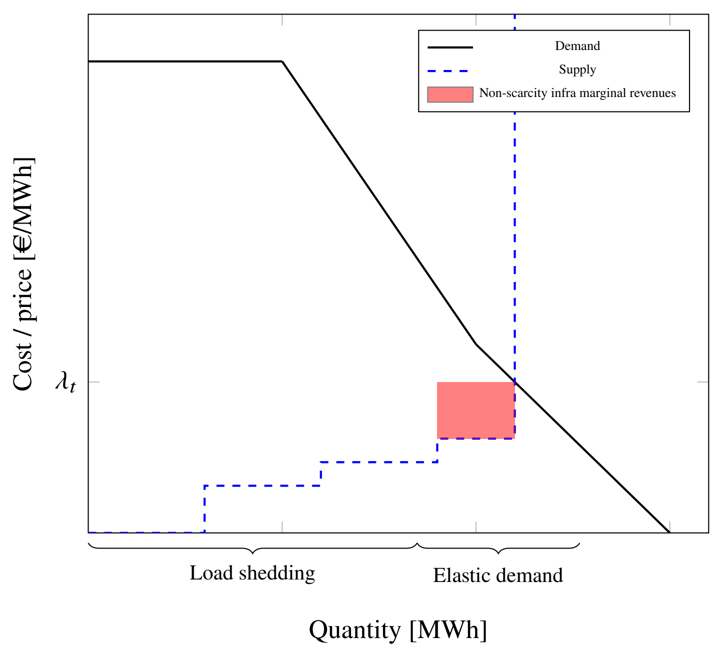

Figure 9: Illustration of non-scarcity infra-marginal revenues (shaded area) in the presence of elastic demand. The electricity price for this time step, λ_t, is set by the elastic portion of the demand.

Recall from Appendix 8.3 that the variable x in Eq. (7) can be interpreted as the non-scarcity

[13] Since if $\lambda_{st} = C_{\hat{g}}^{var}$ we just have a zero term.

infra-marginal revenues of the peaking generator. Figure 9 illustrates this for the case of elastic demand. These revenues are infra-marginal since the peaking generator is operating at full capacity and hence the price is set by the elastic portion of the demand and not the marginal cost of the generator (in which case it would simply recover its operational costs).

The peaking generator could in principle recover its fixed costs (see Eq. (37)) entirely through time steps in which this occurs. In this case there would be no load shedding and hence no adequacy issue to deal with, since consumers voluntarily reduce their demand when supply is operating at full capacity. This is in line with theoretical analyses of electricity markets and the missing money problem (see e.g. (Höschle, 2018)) though in practice a large volume of elastic demand may be required (Kaminski et al., 2021).

8.5 Mathematical proof that $\phi_t' = -1$ for Problem (9) when $w(\phi)$ is convex and strictly increasing in ϕ

This section elaborates on the model and intuitive proof given in Section 3. We begin by deriving the necessary condition for this proof in Appendix 8.5.1. This is followed by a derivation of the optimality conditions of Problem (9) in Appendix 8.5.2. The final three sections prove using the optimality conditions that the necessary condition is satisfied for the case where the ELR is energy constrained, power constrained and both energy and power constrained.

8.5.1 Necessary condition for $\phi' = -1$

Recall that we want to prove that the optimal dispatch for the problem (9) leads to $\phi_t' = -1$. This is equivalent to proving the following:

$$\lim_{k \longrightarrow 0} d_t^k - d_t = 0 \quad \forall t \in \mathcal{T}^E \tag{38}$$

where the superscript k denotes variables belonging to the problem (9) in which an amount of firm capacity k has been added. In other words, adding a marginal amount of firm capacity does not change the ELR dispatch.

The condition 38 is obtained by noting that adding firm capacity k is equivalent to reducing the demand net of generation Z_t by k:

$$
\begin{aligned}
\phi_t^k - \phi_t &= (Z_t - d_t) - (Z_t - k - d_t^k) \\
\Delta\phi_t &= -k + (d_t^k - d_t) \\
\frac{\Delta\phi_t}{k} &= -1 + \frac{(d_t^k - d_t)}{k} \\
\phi_t' &= \lim_{k \longrightarrow 0} \frac{\Delta\phi_t}{k} \\
\phi_t' &= \lim_{k \longrightarrow 0} \left(-1 + \frac{(d_t^k - d_t)}{k}\right)
\end{aligned} \tag{39}
$$

Eq. (39) clearly implies that for $\phi_t' = -1$ Eq. (38) must hold. Another way of interpreting this condition is that adding firm capacity k must not change the dispatch of ELR.

Having derived this condition we move on to deriving the optimality conditions of Problem (9).

8.5.2 Optimality conditions of Problem (9)

The Lagrangian of problem (9) is given by:

$$
\begin{aligned}
\mathcal{L} = &\sum_{t \in \mathcal{T}^E} \int_0^{\phi_t} w(\phi) d\phi + \sum_{t \in \mathcal{T}^E} \lambda_t \cdot (d_t + \phi_t - Z_t) \\
&+ \mu \cdot \left(\sum_{t \in \mathcal{T}^E} d_t - E\right) + \sum_{t \in \mathcal{T}^E} \alpha_t \cdot (d_t - P) \\
&+ \sum_{t \in \mathcal{T}^E} \gamma_t \cdot (-d_t) + \sum_{t \in \mathcal{T}^E} \epsilon_t \cdot (-\phi_t)
\end{aligned} \tag{40}
$$

At optimality the following holds:

$$
\frac{\partial \mathcal{L}}{\partial \phi_t} = w(\phi_t) + \lambda_t - \epsilon_t = 0 \quad t \in \mathcal{T}^E \tag{41}
$$

$$
\frac{\partial \mathcal{L}}{\partial d_t} = \lambda_t + \mu + \alpha_t - \gamma_t = 0 \quad t \in \mathcal{T}^E \tag{42}
$$

This is how Eq. (10) (reproduced below) was obtained. The corresponding complementarity conditions are summarised in Table 1.

$$\mu + \alpha_t + \epsilon_t = w(\phi_t) + \gamma_t \quad t \in \mathcal{T}^E \tag{43}$$

8.5.3 Proof when Energy Limited Resource is only energy constrained

Let's assume for the time being that the ELR is only energy limited ($\mu > 0$) and not power limited ($\alpha_t = 0$). Recall from Section 3.3 that for timesteps when ELR is dispatched ($\gamma_t = 0$) that $\mu = w(\phi_t)$ (Eq. (11)). When it is not dispatched we obtain the following:

$$\mu = w(\phi_t) + \gamma_t \quad t \in \mathcal{T}^E \tag{44}$$

Let the subscript D denote the case where the ELR is dispatched and N when it is not. We can then equate Eq. (11) and Eq. (44):

$$w(\phi_{t_D}) - w(\phi_{t_N}) = \gamma_{t_N} \tag{45}$$

Since $\gamma_t > 0$ the following inequalities hold:

$$w(\phi_{t_D}) > w(\phi_{t_N}) \tag{46}$$

$$\phi_{t_D} > \phi_{t_N} \tag{47}$$

$$\phi_{t_D} > Z_{t_N} \tag{48}$$

Put differently, load shedding during timesteps in which ELR is dispatched is less than in timesteps in which it is. In addition we note that when ELR is not dispatched then the energy balance specifies that $\phi_{t_N} = Z_{t_N}$.

To prove that we satisfy Eq. (38), we consider two distinct timesteps t_1 and t_2 in which ELR is being dispatched. We know that the load shedding in these timesteps is the same from Eq. (11), so we can equate their energy balances:

$$Z_{t_1} - d_{t_1} = Z_{t_2} - d_{t_2} \tag{49}$$

We can equally do this for the case in which we add a small amount of firm capacity k:

$$Z_{t_1} - d_{t_1}^k - k = Z_{t_2} - d_{t_2}^k - k \tag{50}$$

Taking the difference between Eq. (49) and (50) gives:

$$d_{t_1} - d_{t_1}^k = d_{t_2} - d_{t_2}^k \tag{51}$$

In other words, the difference in dispatch which occurs from adding firm capacity is the same in all timesteps. This difference in dispatch could be positive, negative or zero. If it is positive or negative then total amount of energy dispatched by the ELR would increase or decrease respectively. If it increased, it would violate the ELR's energy limits. If it decreased then the ELR would not be energy constrained, in which case no load shedding occurs at all. The only possibility remaining is that the difference in dispatch is zero, i.e. $d_t - d_t^k = 0$. In other words, adding firm capacity does not change the ELR dispatch and hence we have satisfied Eq. (38).

8.5.4 Proof when Energy Limited Resource is only power constrained

Consider the timesteps in which the ELR is not power constrained, $t \in \mathcal{T}^E \setminus T^{PC}$, in which case $\alpha_t = 0$. Eq. (10) then reduces to:

$$\epsilon_t = w(\phi_t) + \gamma_t \quad t \in \mathcal{T}^E \tag{52}$$

Since $w(\phi_t) > 0$ for $\phi_t \geq 0$, $\epsilon_t > 0$ and therefore $\phi_t = 0$, i.e. no load shedding occurs in these timesteps. It is therefore unnecessary to investigate whether $\phi' = -1$ for this case.

Let us now turn our attention to timesteps in which ELR is power constrained, $t \in T^{PC}$. For these timesteps $\alpha_t > 0$ and therefore $d_t = P$. From the energy balance we obtain $\phi_t = Z_t - P$. Since ELR is being dispatched, $\gamma_t = 0$. We will also ignore the edge case where $Z_t = P$, implying that $\phi_t > 0$ and $\epsilon_t = 0$. Finally we obtain the following:

$$\alpha_t = w(Z_t - P) \quad t \in \mathcal{T}^E \setminus T^{PC} \tag{53}$$

To summarise for a purely power constrained ELR at each time step the store either discharges up to the point that no load shedding occurs or until it reaches its power limit.

Recall that we want to prove Eq. (38) for the cases where $\phi_t > 0$. This is trivially proven:

$$d_t^k - d_t = P - P = 0 \quad t \in T^{PC} \tag{54}$$

This is unsurprising, as a purely power constrained storage acts as a conventional generator during times of scarcity for which it is well known that $\phi' = -1$.

8.5.5 Proof when Energy Limited Resource is both energy and power constrained

We begin by noting that the proof in Appendix 8.5.3 that $\phi' = -1$ for timesteps in which ELR is not power constrained still holds. We therefore only need to consider the timesteps in which it is additionally power constrained, $t \in^{PC}$. For these timesteps $d_t = P$ and so $\gamma_t = 0$. Ignoring the edge case where $\phi_t = 0$ we have that $\phi_t > 0$ and so $\epsilon_t = 0$, giving:

$$\mu = w(\phi_t) - \alpha_t \tag{55}$$

Comparing condition (11) and Eq. (55) we can deduce that $\phi_{t_D} > \phi_{t_N}$, which is the expected result that load shedding is greater in timesteps in which the ELR is power constrained. Since $d_t = P$, Eq. (54) can be recycled to prove that the condition (38) holds for these timesteps.

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